Control Systems Laboratory (EE 3321) — Experiment 2

Modeling and Simulation

I. Overview of Experimental Procedure

In this experiment, the student will create a model of a tank using physical equations. Next, they will simulate the model using MATLAB Simulink to study the dynamics of the tank level in response to changes in inflow. Following this, the student will derive the linearized model and perform some open loop experiments to compare the dynamics of the nonlinear and the simplified linear systems around the desired operating point.

II. Plant Description

In this experiment, we will analyze the Coupled-Tank plant. The hardware for this plant is available in the lab, allowing you to examine the actual system you will be working with. This plant will be used for hardware implementation in the final two experiments. Here, we will conduct software simulations for this plant.

A schematic of the Coupled-Tank plant is represented in Figure 2.1, below. As illustrated in Figure 2.1, the positive direction of vertical level displacement is upwards, with the origin at the bottom of each tank (i.e. corresponding to an empty tank).

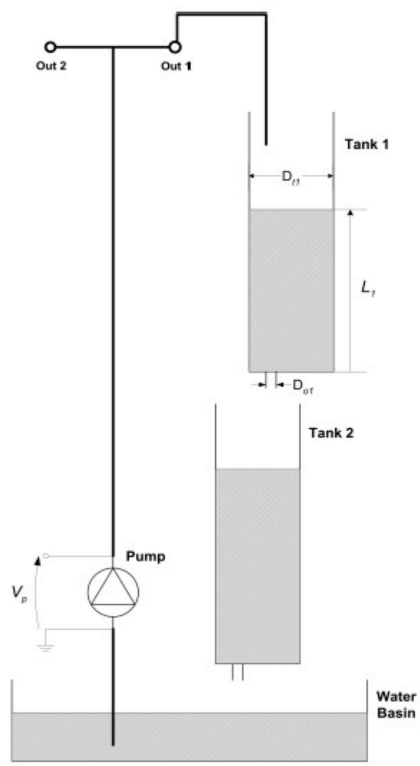


Figure 2.1: Schematic of Coupled Tank.

The system's two water tanks are made of Plexiglas tubes of uniform cross section. The Coupled-Tank pump is a gear pump composed of a DC motor rated for 12 V continuous and 22 V peak with heat radiating fins.

Each tank's actual liquid level is measured through a pressure sensor. Such a level sensor is located at the bottom of each tank and provides linear level readings over the complete liquid vertical level. In other words, the sensor output voltage increases proportionally to the applied pressure. Its output measurement is processed through a signal conditioning board and made available as 0 to 5V DC signal. Moreover, calibration of each pressure sensor's offset and gain potentiometers is required to keep level measurements consistent with the type of liquid used in the coupled-tank experiment.

III. Nonlinear Equation of Motion (EOM)

To derive the mathematical model of your Coupled-Tank system in Figure 2.1, it is reminded that the pump feeds into Tank 1 and that Tank 2 is not considered at all. Therefore, the input to the process is the voltage to the pump V_p and its output is the water level in tank 1, L_1, (i.e., top tank). The obtained Equation of Motion, EOM, should be a function of the system's input and output, as previously defined.

Therefore, you should express the resulting EOM under the following format:

$$\frac{\partial L_1}{\partial t} = f(L_1, V_p) \tag{2.1}$$

Where f denotes a function.

In deriving the Tank 1 EOM the mass balance principle can be applied to the water level in tank 1, i.e.

$$A_{t1}\frac{\partial L_1}{\partial t} = F_{i1} - F_{o1} \tag{2.2}$$

Where A_{t1} is the area of Tank1. F_{i1} and $F_{o1}$are the inflow rate and outflow rate, respectively. The volumetric inflow rate to tank 1 is assumed to be directly proportional to the applied pump voltage, such that:

$$F_{i1} = K_p V_p \tag{2.3}$$

Applying Bernoulli's equation for small orifices, the outflow velocity from tank 1, v_{01}, can be expressed by the following relationship:

$$v_{o1} = \sqrt{2gL_1} \tag{2.4}$$

IV. EOM Linearization

Real systems often exhibit nonlinearity. However, to analyze system behavior or design a controller, we often work with linear systems. In control theory, we use Taylor series expansion to linearize a nonlinear system around an equilibrium point. This equilibrium point is where we want our system to settle in steady state. Therefore, we can find the equilibrium point by setting all the derivatives of the nonlinear differential equation to zero.

The nonlinear EOM of tank 1 should be linearized around a quiescent point of operation. By definition, static equilibrium at a nominal operating point (V_{p0}, L_{10}) is characterized by the Tank 1 level being at a constant position L_{10} due to a constant water flow generated by constant pump voltage V_{p0}.

In the case of the water level in tank 1, the operating range corresponds to small departure heights, L_{11}, and small departure voltages, V_{p1}, from the desired equilibrium point (V_{p0}, L_{10}). Therefore, L_1 and V_p can be expressed as the sum of two quantities, as shown below:

$$L_1 = L_{10} + L_{11}, \;\; V_p = V_{p0} + V_{p1} \tag{2.5}$$

The obtained linearized EOM should be a function of the system's small deviations about its equilibrium point (V_{p0}, L_{10}). Therefore, one should express the resulting linear EOM under the following format:

$$\frac{\partial L_{11}}{\partial t} = f(L_{11}, V_{p1}) \tag{2.6}$$

Where f denotes a function.

V. Coupled Tank Model Parameters

The table below, lists and characterizes the main parameters associated with the two-tank specialty plant. Some of these parameters can be used for mathematical modelling of the Coupled-Tank system as well as to obtain the water level's Equation of Motion (EOM).

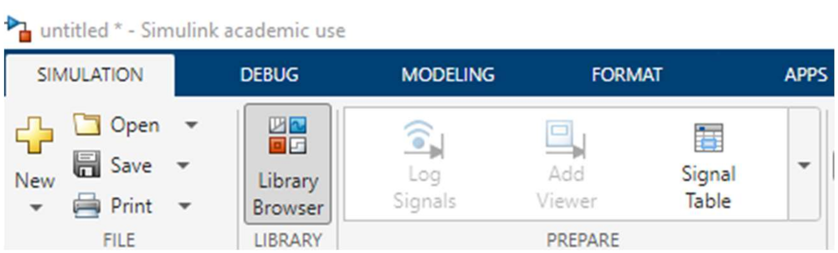
Symbol	Description	Value	Unit
K_P	Pump Flow Constant	3.3	$cm^3/s/V$
V_{Pmax}	Pump Maximum Continuous Voltage	12	V
V_{Ppeak}	Pump Peak Voltage	22	V
D_{Out1}	Out 1 Orifice Diameter	0.635	cm
D_{Out2}	Out 2 Orifice Diameter	0.47625	cm
L_{1max}	Tank 1 Height (i.e. Water Level Range)	30	cm
D_{t1}	Tank 1 Inside Diameter	4.445	cm
K_{L1}	Tank 1 Water Level Sensor Sensitivity (Depending on the Pressure Sensor Calibration).	6.1	cm/V
L_{2max}	Tank 2 Height (i.e. Water Level Range)	30	cm
K_{L2}	Tank 2 Water Level Sensor Sensitivity (Depending on the Pressure Sensor Calibration).	6.1	cm/V
L_{2max}	Tank 2 Height (i.e. Water Level Range)	30	cm
V_{bias}	Tank 1 and Tank 2 Pressure Sensor Power Bias	+/-12	V
P_{range}	Tank 1 and Tank 2 Sensor Pressure Range	0 - 6.89	kPa
D_{So}	Small Outflow Orifice Diameter	0.31750	cm
D_{Mo}	Medium Outflow Orifice Diameter	0.47625	cm
D_{Lo}	Large Outflow Orifice Diameter	0.55563	cm
g	Gravitational Constant on Earth	981	cm/s^2

VI. Simulink Use

MATLAB Simulink is a graphical programming environment for modeling, simulating, and analyzing multi domain dynamic systems. Its primary interface is a graphical block diagramming tool and a customizable set of block libraries. To access Simulink, click the Simulink button from the MATLAB home menu. Create a new file.

There are two primary ways to add blocks to a Simulink model.

1. In the top left corner of the Simulink "Simulation" tab, click the Library browser, which holds the library of usable blocks. A block can be added to the simulation by dragging it to the workspace.



2. Blocks can be searched for by double clicking the workspace.

There are several key blocks that are useful for dynamic simulations in Simulink.

- Input blocks: These blocks are found under the "sources" tab in the library browser. They are the inputs to your simulation. Examples include: step, ramp, constant, and signal generator.

- Continuous time blocks: These blocks are found under the “continuous” tab in the library browser. These blocks are related to differential equations and are the workhorse of dynamic simulations. Examples include: Transfer functions, integrators, and derivatives.
- Singal routing blocks. Located in “signal routing”, these blocks are used to manipulate the signal flows in the simulation. Examples include mux, switches, and selectors.
- Math operation blocks: Located in “math operations” these blocks are used to manipulate the signals in the simulation. Examples include: gain, sum, square, and square root.
- Output blocks: Located in “sinks” these blocks are used to present results in a human-readable format. Examples include: scope and display.

VII. Experimental Procedure

1) Derive the Equation of Motion characterizing the dynamics of tank 1. Is the tank 1 system's EOM linear?

   Hint: The outflow rate from tank 1, F_{o1}, can be expressed by: $F_{o1} = A_{o1} v_{o1}$
2) Find the nominal pump voltage V_{p0} at system's static equilibrium. By definition, static equilibrium at a nominal operating point (V_{p0}, L_{10}) is characterized by the water in tank 1 being at a constant position level L_{10} due to the constant inflow rate generated by V_{p0}. (Assume $L_{10} = 15cm$).
3) Simulate the behavior of the tank 1 around the equilibrium point using basic components in MATLAB Simulink such as gain, step, sum, integrator, scope, and so on.
4) Linearize tank 1 water level's EOM found in Problem 1 about the quiescent operating point (V_{p0}, L_{10}).
5) Simulate the behavior of the linearized tank 1 around the equilibrium point using basic components in MATLAB Simulink such as gain, step, sum, integrator, scope, and so on.
6) Compare the behavior of the nonlinear and linear models of Tank 1 around the equilibrium point. Conduct three experiments using a sinusoidal input with amplitudes of 1, 3, and 10.